2025

HATT Ch 3 Practice Name: Key Block: Seat:

1. (6 points) Consider the graph below

   (a) Is this a graph of a function?

   yes, passes vert. line test

   (b) On what interval is it increasing?

   $(-\infty, 10)$

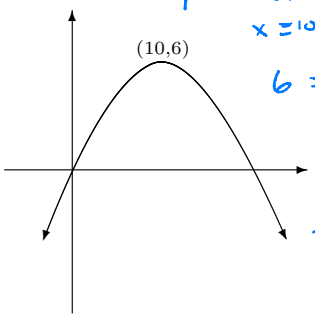
   (c) Where is the maximum?

   6 at (10, 6)

   (d) What is the range?

   $(-\infty, 6]$

   (e) Find an equation which has this graph.



$y = a(x-10)^2 + 6$

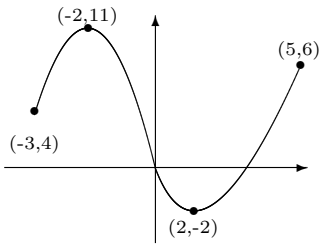
$x = 10 \quad y = 6$

$6 = a(10-10)^2 + 6$

$a = -\frac{6}{100} = -\frac{3}{50}$

$f(x) = -\frac{3}{50}(x-10)^2 + 6$

2. (3 points) Consider the relation whose graph is



   (a) What is the domain?

   $[-3, 5]$

   (b) What is the range?

   $[-2, 11]$

   (c) On what interval(s) is it decreasing?

   $(-3, -2) \cup (2, 3)$

3. (2 points) Use interval notation to describe the domain of $f(x) = \frac{\sqrt{1-x}}{x^2-1}$?

$1 - x \geq 0$

$x^2 - 1 \neq 0$

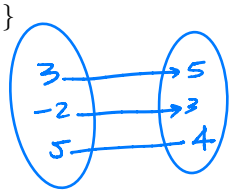
If $1 - x \geq 0$, $x \leq 1$

If $x^2 - 1 \neq 0$ $x \neq \pm 1$

$(-\infty, -1) \cup (-1, 1)$

4. (3 points) Consider the relation

$$\{(3,5), (-2,3), (5,4)\}$$



   (a) Is this a function?

   yes,

   (b) What is the preimage of 3?

   -2

   (c) What is the image of 3?

   5

5. Is $h(x) = \frac{2x}{x^2-1}$ even, odd or neither?

$h(-x) = \frac{-2x}{x^2-1} = -h(x)$ so h is odd (sym. to origin)

6. (4 points) Consider f/g, where $f(x) = x^2$ and $g(x) = |x-1|$

   (a) (2 points) $(f/g)(x) =$ $\frac{x^2}{|x-1|}$

   (b) What is the domain of f/g?

   $(-\infty, 1) \cup (1, \infty)$ or $\{x \in \mathbb{R} \mid x \neq 1\}$

   (c) What is the range of f/g?

   $[0, \infty)$

7. (4 points) Consider $g + f$ where $f(x) = x^2$ and $g(x) = |x-1|$, what is $g + f$?

   (a) (2 points) $(g+f)(x) =$

   $|x-1| + x^2$

   (b) What is the domain of $g + f$?

   $(-\infty, \infty)$

   (c) What is the range of $g + f$?

   $[0, \infty)$

8. (3 points) Recall the "slope" or "average rate of change" or "difference quotient" of a function can be computed in various ways:

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h)-f(x)}{h} = \frac{f(b)-f(a)}{b-a}$$

Let $f(x) = 2x^2 - x$

(a) What is the average rate of change of f from 0 to 1?

$f(0) = 0$ and $f(1) = 1$

$$\text{slope} = \frac{f(1)-f(0)}{1-0} = \frac{1-0}{1-0} = 1$$

(b) What is the average rate of change of f from 1 to x?

$$\frac{f(x)-f(1)}{x-1} = \frac{2x^2-x-1}{x-1}$$

$$= \frac{(2x+1)(x-1)}{(x-1)}$$

$$= 2x+1 \quad (\text{but } x \neq 1)$$

(c) What is the average rate of change of f from x to $x+h$?

$f(x+h) = 2(x+h)^2 - (x+h)$

$= 2(x^2+2hx+h^2) - x - h$

$= 2x^2 + 4hx + h^2 - x - h$

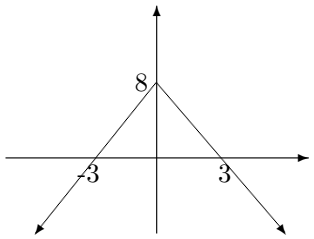
$f(x) = 2x^2 - x$

$$\frac{f(x+h)-f(x)}{x+h-x} = \frac{2x^2+4hx+h^2-x-h-2x^2+x}{h}$$

$$= \frac{4hx+h^2-h}{h}$$

$$= 4x+h-1$$

9. (4 points) Consider



(a) What is the domain?

$(-\infty, \infty)$

(b) What is the range?

$(-\infty, 8]$

(c) Find an equation that has a graph like this.

$y = a|x| + 8$

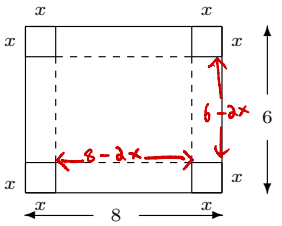
using $(-3, 0)$

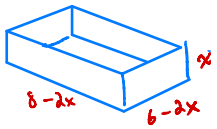
$0 = a|-3| + 8$

$a = -\frac{8}{3}$

$f(x) = -\frac{8}{3}|x| + 8$

10. (4 points) An open box with a rectangular base is to be made with a piece of cardboard 6 in by 8 in. by cutting out a square from each corner and turning up the sides (see figure). How large should x be to have the most volume?





(a) Make a function $V(x)$ that expresses the volume of the box as a function of x.

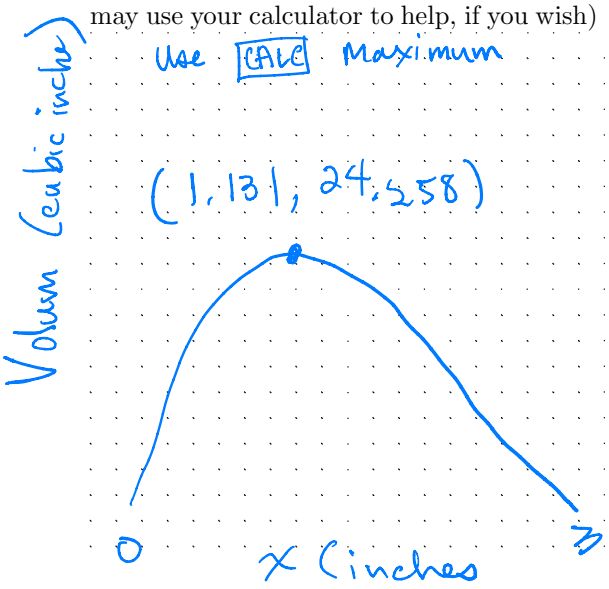
$V(x) = x(8-2x)(6-2x)$

(b) Because of the size of the cardboard, this function needs to have a limited domain. What should be the domain of V?

smallest side is $6-2x = 2(3-x) > 0$ so $\boxed{0 < x < 3}$

(c) Sketch a graph of V over this domain (you may use your calculator to help, if you wish)

Use CALC Maximum



(d) What length x should be chosen to maximize volume V?

1.131 inches

$V(1.131) \approx 24.258$ cubic inches

11. (4 points) A camera company has found that the revenue from sales R (measured in dollars), is a function of the unit price p it charges:

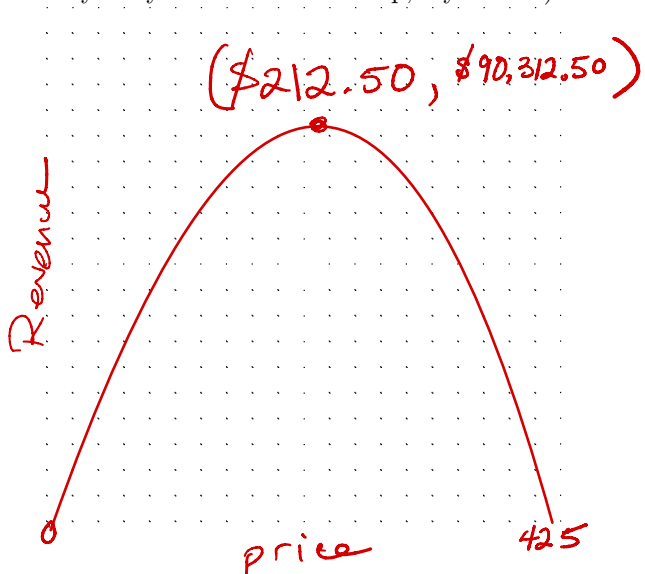
$$R(p) = -2p^2 + 850p = -2p(p-425)$$

(a) If we wish to make more than \$0 in revenue, what is the domain?

$0 < p < \$425$

$p \in (0, 425)$

(b) Sketch a graph of R over this domain (you may use your calculator to help, if you wish)



(c) What unit price p should be charged to maximize revenue?

Use CALC Max on TI84

12. (6 points) Consider $g(x) = 3 - 2|x+1|$

(a) What family of functions does g belong?

Absolute Value

(b) How much is the horizontal shift (left or right)?

one to the left

(since $x+1=0$ when $x=-1$)

(c) How much is the vertical shift (up or down)?

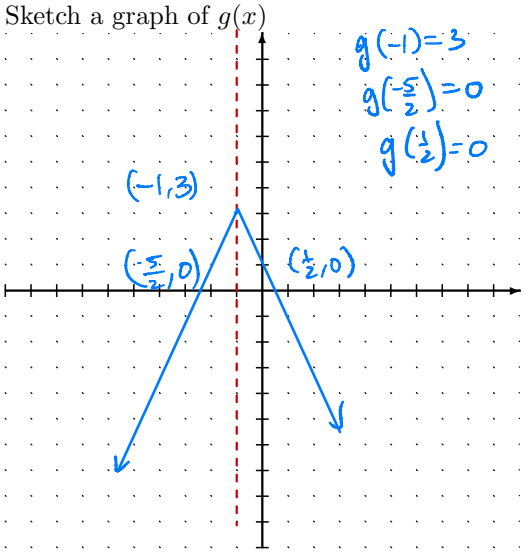
up by 3

(since $3-0=3$ when $x+1=0$)

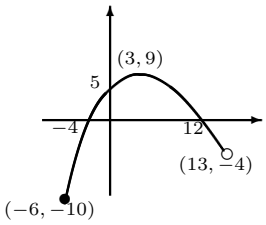
(d) Is there any horizontal of vertical reflection?

not really but there is a reflection with respect to the line $x=-1$

(e) Sketch a graph of $g(x)$



13. (5 points) Consider the graph of $g(x)$ below:



(a) Describe the domain of g in interval notation

$[-6, 13)$

(b) Describe the range of g in interval notation

$[-10, 9]$

(c) Describe the maximum of g

9

$g(3) = 9$

(d) Describe the minimum of g

-10

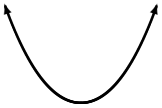
$g(-6) = -10$

(e) Describe all zeros of g

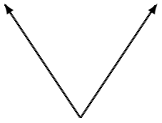
-4 and 12

Since $g(-4) = g(12) = 0$

14. (9 points) Name the function family of each graph.

(a) 

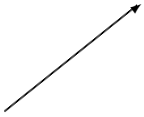
square (eg: $y = x^2$)

(b) 

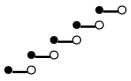
absolute value (eg: $y = |x|$)

(c) 

square root (eg: $y = \sqrt{-x}$)

(d) 

linear (eg: $y = x$)

(e) 

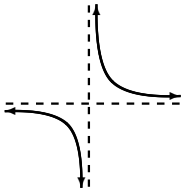
greatest integer (eg: $y = [\![x]\!]$ or $y = \text{int}(x)$)

(f) 

square (eg: $y = -x^2$)

(g) 

cubic (eg: $y = x^3$)

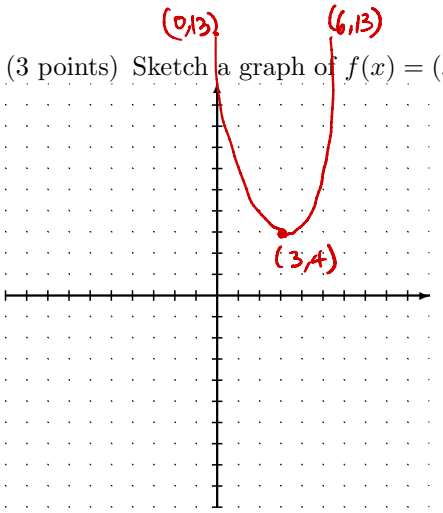
(h) 

reciprocal (eg: $y = \frac{1}{x}$)

(i) 

square root (eg: $y = \sqrt{x}$)

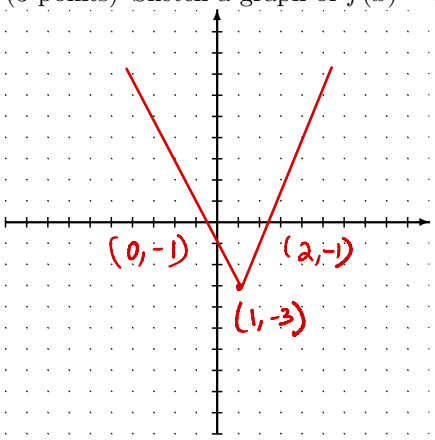
15. (3 points) Sketch a graph of $f(x) = (x-3)^2 + 4$



$f(3) = 4$

$f(0) = 13$

16. (3 points) Sketch a graph of $f(x) = 2|x-1| - 3$

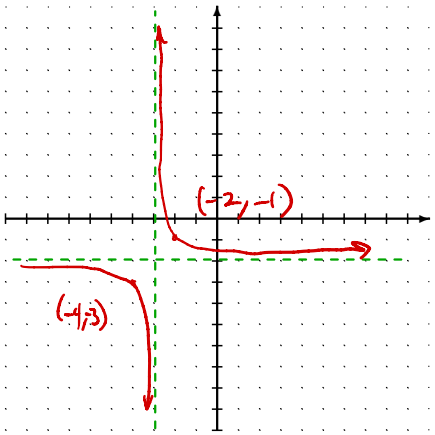


$f(1) = -3$

$f\left(\frac{5}{2}\right) = 0$

$f(0) = -1$

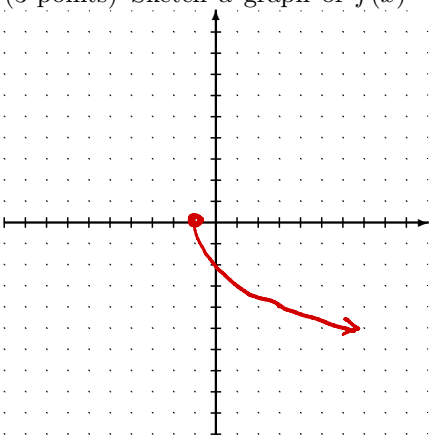
17. (3 points) Sketch a graph of $f(x) = \dfrac{1}{x+3} - 2$



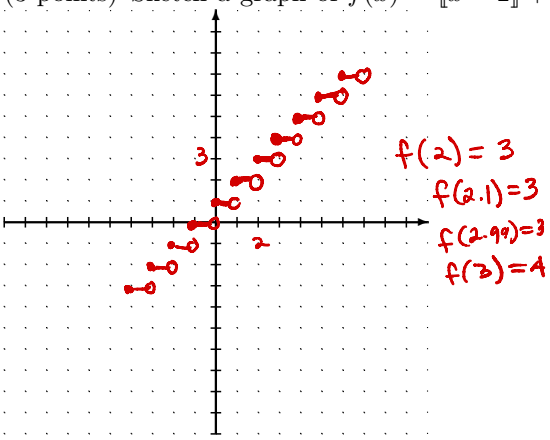
$f(-2) = -1$

$f(-4) = -3$

18. (3 points) Sketch a graph of $f(x) = -2\sqrt{x+1}$



19. (3 points) Sketch a graph of $f(x) = [\![x-2]\!] + 3$



20. (4 points) First class mail rates for letters in 2010:

Weight not over	Price
1	.44
2	.61
3	.78
3.5	.95

(a) Define a piece-wise function $P(w)$ that describes price as a function of weight w.

$$P(w) = \begin{cases} 0.95, & 3 < w \leq 3.5 \\ 0.78, & 2 < w \leq 3 \\ 0.61, & 1 < w \leq 2 \\ 0.44, & 0 < w \leq 1 \end{cases}$$

(b) Graph your function